Notes from Text

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1 Introduction

This introduction will provide some review of the methods and problems we've covered in class. There are a couple of terms that I use that you might not be familiar with. I'll discuss them here. When someone says, "pairwise," in the context of a list (or set), it usually means another list (or set) that contains *all* the *possible* pairs, but typically without concern to the *order* of the pairs. Let's see an example.

Example 1 Let's assume we have list of letters and number $X = (a, b, 1, 5)$. Here's a pairwise set $((a, a), (a, b), (a, 1), (a, 5), (b, b), (b, 1), (b, 5), (1, 1), (1, 5), (5, 5))$. Although you don't explicitly *see* (b, a), pairwise means (a, b) is the *same* as (b, a). You can actually treat pairwise as a kind of multiplication of lists (or sets). "Pairwise" means you want to associate every element of a list with every other element. A couple of conditions to consider are, "Do you want to compare elements that are identical?" and "Is the order of this kind of multiplication important?" For our purposes, we will *not* worry about comparing elements that are identical with themselves. ■

1.1 Weighted Ranking

Sometimes in problem solving you have a number of solutions *already* prepared–you don't have to find or discover them. In this kind of scenario, you must figure out how to identify the best solution among those you have. Typically people choose different solutions for different reasons. For example, in picking a computer monitor you like one because of its price, another because it high-definition, another because it'd be great for your computer system. We claim that the optimal solution cannot be made by using *different* criteria–reasons to choose a solution–to *different* solutions. You must be object and honest and apply *all* the criteria to *all* the solutions. Weighted ranking is a means of doing this. Here's a general definition of weighted ranking:

Definition 1 Weighted Ranking is a method of *uniformly* identifying and applying criteria that can used to rank solutions to a problem. Here are the steps:

1. Generate the problem statement
2. Generate solutions to the problem statement
3. Generate criteria (make sense of the criteria)
4. Rank criteria pairwise
5. Apply the *most* important criteria pairwise to the solutions in 2.

□

We'll do a couple of examples to show how weighted ranking is done. The first example will be a rather simplified so the steps will be clear to you. The second example will expose more of the subtle elements of this process.

Example 2 This is a simple problem concerning making meal for some friends.

1. Problem statement: Make lunch for five friends. The CSA has these constraints
   - One friend is vegetarian
   - You have a limited budget of twenty dollars
   - You only have a large toaster oven that can broil, bake, and toast
   - dormitory food isn't tastey
2. Solutions
   (a) Sandwiches,
   (b) Beans and rice
   (c) Fast food
   (d) Dormitory (Dorm) Food
3. Criteria
   (a) Fast food is quick.
   (b) Beans and rice will take a long time using the toaster machine, but it's vegetarian.
   (c) Sandwiches are easy, but you have to go to the grocery store and purchase everything. You probably won't have enough money for condiments–and you're not even sure which condiments are preferred
   (d) The meal should be relatively easy to prepare and serve, since your dorm room is small
   (e) You don't have any large pots and pans
   (f) There's a large discount store with pots and pans nearby
   (g) Use the money and eat at the dormitory cafeteria

   Now let's make "sense of the criteria." This process is the most difficult. We won't discuss *formally* what you should do, but rather be sensible in eliminating, combinining or splitting criteria. You should take into account the problem statement and CSA. You only have twenty dollars, so if you purchase pots and pans, you'll probably not enough money to complete the meal–so we might as well remove this as a criteria. To cook beans and rice, even though it'd be vegatarian and satisfy everyone, it's unlikely this is viable option since you need pots and pans, and probably something better than a toaster. So, after some making sense of the criteria we can change *both* the solutions available and criteria end up with these (version 2)
4. Solutions (version 2)
   (a) Sandwiches
   (b) Fast food
   (c) Dorm Food
5. Critera (version 2)
   (a) Fast food is quick (quick)
   (b) Sandwiches are easy, but you have to go to the grocery store and purchase everything. You probably won't have enough money for condiments–and you're not even sure which condiments are preferred (easy)
   (c) The meal should be relatively easy to prepare and serve, since your dorm room is small (limited space)

Now we can do a pairwise comparsion of the three criteria. We'll write $X > Y$ if X is more important than Y. Bear in mind, this is an *example* and that you, the reader, might disagree with my choices to illustrate this problem.

Pairwise 1 quick *vs.* easy

<table>
<tr><th>Critera</th><th>Comparison</th><th>Count</th></tr>
<tr><td>quick</td><td>quick>easy</td><td>|</td></tr>
<tr><td>easy</td><td></td><td></td></tr>
<tr><td>limited space</td><td></td><td></td></tr>
</table>

This means I (the author) believe quickness is more important than easiness. So I put a tally | in the one that is more important. You might disagree–this is part of the challenge of making sense of *belief.*

Pairwise 2 quick *vs.* limited space

<table>
<tr><th>Critera</th><th>Comparison</th><th>Count</th></tr>
<tr><td>quick</td><td>quick>limited space</td><td>||</td></tr>
<tr><td>easy</td><td></td><td></td></tr>
<tr><td>limited space</td><td></td><td></td></tr>
</table>

I think that quickness is more important than space.

Pairwise 3 easy *vs.* limited space

<table>
<tr><th>Critera</th><th>Comparison</th><th>Count</th></tr>
<tr><td>quick</td><td></td><td>||</td></tr>
<tr><td>easy</td><td>easy>limited space</td><td>|</td></tr>
<tr><td>limited space</td><td></td><td></td></tr>
</table>

I think that easiness is more important than space.

6. Now, let's assume the two most important criteria are the most important–for a real problem, you might take five or twelve–this would be determined by the context. We now do a pairwise comparison of the solutions *weighted* by the criteria. Let's create the table:

<table>
<tr><th></th><th colspan="2">Critera</th></tr>
<tr><th>Solutions</th><th>quick ||</th><th>ease |</th></tr>
<tr><td>Sandwiches</td><td>|</td><td></td></tr>
<tr><td>Fast Food</td><td></td><td>|</td></tr>
<tr><td>Dorm Food</td><td>|</td><td>|</td></tr>
</table>

So, for Sandwiches there are two tallies || so, this gets two. For Fast food there is one tally | so this gets one. For Dorm food there are two tallies || in quick and one tally in ease so this is $2 \times 2 + 1 = 5$.

It looks like Dorm Food is the right choice since it has the greatest value

■

2 Stuff to Know

1. Vocabulary
   (a) *satisficing* is a contraction between "suffice" and "satisfy" and means the first acceptable solution is the one taken.
   (b) *truth table* is an exhaustive listing of all possible inputs and output for a formula or sentence
   (c) *disparate* means "different"
   (d) *syntax* the rules of how to build things–grammar is sytax for language for example
   (e) *semantics* the meaning (or evaluation) of a sentence or formula. In Propositional Logic, the semantics will be either `T`, `F` or `Bug`.
2. The five techniques used to identify a problem statement is
   (a) *restatement* this is where one paraphrases the statement
   (b) *negation* You form the negation (or opposite) of the statement
   (c) *broaden* You include *more* solutions by broadening the statement
   (d) *narrow* You exclude solutions by narrowing the statement–it becomes more focused
   (e) *treasure hunt* You keep asking, "Why?" continually until you're satisfied.

   As an example, consider the statement, "Increase my GPA."

   (a) *restatement* "Improve my grades."
   (b) *negation* "How do I descrease my GPA?"
   (c) *broaden* "How do I become a better student," or "How can I learn more?"
   (d) *narrow* "How do I do better in I101?" or "How do I do better on the next exam?"
   (e) *why* (*i*) "Why should I increase my GPA?" (*ii*) "Because I want a better resumé" (*iii*) "I want a better job" and so on.

3 Conversions

We've covered conversions, but there are some conversions among powers of 2. We'll look at how to convert between these two.

Example 3 Let's convert 101011_2 to base eight. We need to group by threes:
$\underbrace{101}\,\underbrace{011}\,_2 = 53_8 = 43_{10}$ ■

Example 4 Let's convert 1111011_2 to base eight. We need to group by threes:
$\underbrace{1}\,\underbrace{111}\,\underbrace{011}\,_2 = 173_8 = 123_{10}$ ■

Example 5 Let's convert 101011_2 to base hexadecimal. We need to group by fours:
$\underbrace{10}\,\underbrace{1011}\,_2 = 2B_{16} = 53_8 = 43_{10}$ ■

Example 6 Let's convert 1111011_2 to base hexadecimal. We need to group by fours:
$\underbrace{111}\,\underbrace{1011}\,_2 = 7B_{16}$ ■

Example 7 Let's convert $B6_{16}$ to base 2. We need to only *expand*:
$\underbrace{B}\,\underbrace{6}\,_{16} = \underbrace{1011}\,\underbrace{0110}\,_2 = 10110110_2$ ■

Example 8 Let's convert $B6_{16}$ to base 8. We need to *first* expand to base 2:
$\underbrace{B}\,\underbrace{6}\,_{16} = \underbrace{1011}\,\underbrace{0110}\,_2 = 10110110_2$ then group by threes: $\underbrace{10}\,\underbrace{110}\,\underbrace{110}\,_8 = 266_8 = 182_{10}$ ■

Example 9 Let's convert 27_8 to base 16. We need to only *expand*:
$\underbrace{1}\,\underbrace{0111}\,_8 = 17_{16} = 23_{10}$ ■

3.1 Converting from decimal to base X

Since we haven't learned a formal programming language, I'll show you a couple of examples, but the basic algorithm is this: To convert a number in Y_b in base b to a number in base X, you keep dividing until the quotient is zero and then read the remainders bottom-up. I'll show you a couple of examples.

- Suppose I want to convert 23_{10} to base 2

Number	Quotient	Remainder
$23_{10}/2$	11	1
$11_{10}/2$	5	1
$5_{10}/2$	2	1
$2_{10}/2$	1	0
$1_{10}/2$	0	1

We read bottom up $10111_2 = 23_{10}$.

- Suppose I want to convert 38 decimal to base 4.

Number	Quotient	Remainder
$38_{10}/4$	9	2
$9_{10}/4$	2	1
$2_{10}/4$	0	2

We read bottom up,

$$212_4 = 2 \times 4^2 + 1 \times 4^1 + 2 \times 4^0 = 2(16) + 1(4) + 2(1) = 32 + 4 + 2 = 38_{10}$$

- Now let's convert 123_4 to base 10. This is simply expansion:

$$1 \times 4^2 + 2 \times 4^1 + 3 \times 4^0 = 1(16) + 2(4) + 3(1) = 16 + 8 + 3 = 27$$