INFO I201
Homework 5
Assigned 05/31/2007, Due 06/05/2007.

1. You’ve been hired as a consultant to create a database for Riverzon Books. They ask for three relations, Book, Customer, and Purchases. The Book relation should have three attributes: author, title, and price. The Customer relation should have the customer’s First and Last name and a Customer ID. The Purchases relation should have three attributes: book title, Customer ID, and Date of Purchase.

When you’ve completed creating these three relations, add the following data to the tables: The book Harry Potter is written by J.K. Rowling and is $17.99. Jane Austen wrote Pride and Prejudice, and it sells for $9.34. Memo Dalkilic has written a book The Malamute and You, and it’s price is $40.00 (it’s selling very well). You have three customers: Unice Zipmond whose customer ID is A114, Erik Runkinorder whose customer ID is B244, Jeff Graymousen whose customer ID is C766. There has been two purchases made: Unice purchased the Harry Potter book on May 5, 2007 12:04 pm. Unice also purchased Pride and Prejudice on April 13, 2006 1:32 pm.

(a) Create the a database that contains these tables.

(b) Write an SQL statement that finds
   i. all the books that cost less than $10.00. What will your query give as answer in this instance?
   ii. all the book titles for CID (customer ID) B244.

SQL is based on predicate logic. The above queries can be expressed as:

\[ Q_1 = \{ b | b \in \text{Book} \land b.\text{Price} \leq 10.00 \} \]  \hspace{1cm} \text{This says the set } Q_1 \text{ contains the tuples in the Book relation that have prices less than ten dollars.}

\[ Q_2 = \{ p.\text{Title} | p \in \text{Purchases} \land p.CID = \text{"B244"} \} \]  \hspace{1cm} \text{This says the set } Q_2 \text{ contains all the titles that the customer with CID B244 purchased. Compare your queries with the logic.}

(c) Extra Credit! Write an SQL query to find the prices of the books Unice purchased
   \hspace{1cm} \text{(hint: you’ll need to multiply the relations)}

2. Assume for this question that \( A = \{a, b, c\} \), \( B = \{\alpha, \beta, \gamma\} \), \( C = \{A, B, a, b, \alpha\} \), and \( D = \{c, \gamma\} \). What do the following statements return?

(a) \( A \subset C \)
(b) \( A \in C \)
(c) \( A \cap C \)
(d) \( D - B \)
(e) \( P(D) \)
(f) \( P(\emptyset) \)
(g) $\alpha \in C$
(h) $\{\alpha\} \in B$
(i) $\{\beta, \alpha\} \subset B$
(j) $D \cup \{D\}$
(k) $C \cap \{B, \alpha\}$
(l) $A \times B$
(m) $\langle c, \gamma \rangle \in D \times D$
(n) $\langle \gamma, \gamma \rangle \in B \times D$

3. If a set has $n$ distinct members, the powerset will have how many members?

4. $\forall$ means [every, at least one, no] enviroment makes a formua true.

5. $\exists$ means [every, at least one, no] enviroment makes a formua true.

6. Assume the universe is $U = \{0, 1, 2, 3, 4\}$. There are three predicates:

   $A = \{0, 2, 4\}$

   $B \subseteq U \times U$ and $B = \{\langle x, y \rangle | x \in U \land y \in U \land x \text{ is odd.}\}$

   $C \subseteq U \times U$ and $C = \{\langle 0, 1 \rangle, \langle 2, 3 \rangle, \langle 4, 4 \rangle\}$

And two functions

   $f : U \rightarrow U$ and $f(x) = 0$

   $g : U \rightarrow U$ and $g = \{(0,0), (1,1), (2,2), (3,3), (4,1)\}$

For every variable in the formula, state whether it is free or bound:

(a) $\forall(x_1) A(x_1) \land A(x_2)$

(b) $\exists(y) \forall(y_1) B(a, y_1) \rightarrow C(x_2, y_{23})$

For each statement determine if it is True, False, or Bug. Explain why in either case

(a) $\neg A(2) \rightarrow B(1,0)$

(b) $A(f(2)) \lor B(g(0), g(1))$

(c) $\exists(x_1) A(x_2)$

(d) $\forall(x_1) A(x_1) \land A(f(x_1))$
For the following problems, you’ll be provided with a statement and the environments and the evaluation of the statements in each respective environments.

$$\exists (y_2)B(f(y_2), 0) \land A(y_2)$$

This means that for some environment the formula is true. Let’s look at each of the environments:

<table>
<thead>
<tr>
<th>Environment</th>
<th>Formula</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_1 = [y_2/0]$</td>
<td>$B(f(0), 0) \land A(0)$</td>
<td>F</td>
</tr>
<tr>
<td>$\sigma_2 = [y_2/1]$</td>
<td>$B(f(1), 0) \land A(1)$</td>
<td>F</td>
</tr>
<tr>
<td>$\sigma_3 = [y_2/2]$</td>
<td>$B(f(2), 0) \land A(2)$</td>
<td>F</td>
</tr>
<tr>
<td>$\sigma_4 = [y_2/3]$</td>
<td>$B(f(3), 0) \land A(3)$</td>
<td>F</td>
</tr>
</tbody>
</table>

Is the formula true at least once? Please take time to verify the value.