Assignments: 35%
- Students will complete 4/5 assignments based on algorithms presented in class
- Lab meets in I1 (West) 109 on Lab Wednesdays
  - Lab 0: January 14th (completed)
    - Introduction to Python (No Assignment)
  - Lab 1: January 28th
    - Measuring Information (Assignment 1)
    - Due February 11th
  - Lab 2: February 11th
    - L-Systems (Assignment 2)
    - Due February 25th
  - Lab 3: March 11th
    - Cellular Automata and Boolean Networks (Assignment 3)
Readings until now

- **Class Book**
    - Chapter 8 - Artificial Life
    - Chapter 7, sections 7.1, 7.2 and 7.4 – Fractals and L-Systems
    - Appendix B.3.1 – Production Grammars

- **Lecture notes**
  - Chapter 1: “What is Life?”
  - Chapter 2: “The logical Mechanisms of Life”
  - Chapter 3: Formalizing and Modeling the World
    - posted online @ [http://informatics.indiana.edu/rocha/i-bic](http://informatics.indiana.edu/rocha/i-bic)

- **Papers and other materials**
  - Life and Information
  - Logical mechanisms of life (H400, Optional for I485)

- **Optional**
    - Chapter 1 – Introduction
    - Chapters 5, 6 (7-9) – Self-similarity, fractals, L-Systems
phase or state-space

- Map of variables in time
  - Time is parameter
    - Trajectory (orbit) in state space
  
  \[ X(t) = (x_1(t), x_2(t), x_3(t)) \]

- Continuous (reversible) systems
  - Only one trajectory passes through each point of a state-space
    - State-determined system
    - 2 points on different trajectories will always be on different trajectories
      - Albeit arbitrarily close
  
  - Determinism, strict causality
    - Laplace
  
  - Not true in discrete systems
vector fields in phase-space
where motion leads to volumes of phase space to which the system converges after a long enough time.

Fixed-point behavior (0-dimensional attractor)

Basin of attraction
Volume of the phase-space defined by all trajectories leading into the attractor.
why the attractor behavior?

- Energy dissipation (thermodynamic systems)
  - Friction, thermodynamic losses, loss of material, etc.
  - Volume contraction in phase-space
    - System tends to restrict itself to small basins of attraction
  - Self-organization
    - Dissipative systems (Prigogine)

- Hamiltonian systems
  - Frictionless, no attractors
  - Conservation of energy
  - Ergodicity
Morphogenesis
- development of the structure of an organism or part
  - phenotype develops in time under the direction of the genotype + dynamic constraints
- The process in complex system-environment exchanges that tends to elaborate a system's given form or structure.
- Fischer (1924)
  - Reaction-diffusion equation
    - Propagation of a gene a population
- Nicolas Rashevsky
  - Embryogenesis
- Alan Turing
  - spent the last few years of his life developing his morphogenetic theory and using the new computer to generate solutions to reaction-diffusion systems.


\[
\begin{align*}
\frac{\delta a}{\delta t} &= f(a,b) + D_a \left( \frac{\delta^2 a}{\delta x^2} + \frac{\delta^2 a}{\delta y^2} \right) \\
\frac{\delta b}{\delta t} &= g(a,b) + D_b \left( \frac{\delta^2 b}{\delta x^2} + \frac{\delta^2 b}{\delta y^2} \right)
\end{align*}
\]
Reaction-diffusion model
- Stable tension between production and transformation
  - When balance is disturbed, tension restores balance:
- Metaphor
  - Island populated by cannibals and (celibate) missionaries.
  - Missionaries do not reproduce, but can recruit and die (transform)
  - Cannibals reproduce and die (produce)
  - Two missionaries convert a cannibal, leading to tension between production and transformation

Kele's Science Blog
Reaction-diffusion model
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(energy) landscape metaphor

- Phase-space as landscape
  - State of the system as a drop of water released in hills and valleys

Attractors

Stable

Unstable

Stable

Attractor 1

Attractor 2
types of attractors

- **Attractors**
  - Phase space volume to where dynamical system converges asymptotically over time

- **Fixed point**
  - Steady-state
  - Saddle
    - Stable in a dimension and unstable on another
    - When basins of attraction meet
- Limit cycle
  - Periodic motion
  - Repetitive oscillation among a number of states
    - loop

- 2 values
- 4 values

Types of attractors
- Quasiperiodic attractor
  - Several independent cyclic motions
  - Toroidal attractors
  - Never quite repeat themselves
Strange or chaotic attractors

- Sensitivity to initial conditions
  - If system is released from two distinct, arbitrarily close points on the attractor basin, after sufficient time their trajectories will be arbitrarily far apart from each other

- Deterministic Chaos
  - If we could know the exact initial condition, trajectory would be determined

- Low-dimensional chaos
  - Strange attractors are restricted to small volumes of phase-space
    - More ordered than Hamiltonian or larger limit cycles

- Weak Causality
  - 3-body problem
    - Any slight measurement difference results in very different predictions
  - Butterfly effect
    - Lorenz attractor
Edward Lorenz

- Discovered sensitivity to initial conditions in a simple 3-variable dynamical system
  - A simplified model of weather
  - Convection flows in the atmosphere

\[
\begin{align*}
\frac{dx}{dt} &= \sigma(y-x) \\
\frac{dy}{dt} &= r x - y - xz \\
\frac{dz}{dt} &= xy - bz
\end{align*}
\]
Lorenz attractor
Phasespace
**The logistic map**

- **Demographic model**
  - introduced by Pierre François Verhulst in 1838
- **Continuous state-determined system**
  - Memory of the previous state only
- **Observations**
  - \( X=0 \): population extinct
  - \( X=1 \): Overpopulation, leads to extinction

\[
x_{t+1} = rx_t(1 - x_t)
\]

- **Population size**
  - \( x \in [0,1] \)
  - \( r \in [0,4] \)

- **Reproduction rate**
  - Positive feedback
  - Negative feedback

**quadratic equation**
\[ x_{t+1} = rx_t(1 - x_t) \]
\( x \in [0,1] \)
\( r \in [0,4] \)

\[ x = 0 \lor x = 1 - \frac{1}{r} \]

Fixed-point attractors

\[ f(x) = r x (1 - x) \]
\[ f(x) = x \implies r x (1 - x) = x \implies x(r(x - 1) + 1) = 0 \]

\[ f'(x) = r(1 - 2x), \begin{cases} |f'(x)| < 1 \implies \text{stable} \\ |f'(x)| > 1 \implies \text{unstable} \end{cases} \]
logistic map

\[ r \leq 1 \]

\[ x = 0 \lor x = \frac{1}{1} \]

\[ f'(x) = r(1 - 2x), \begin{cases} |f'(x)| < 1 \Rightarrow \text{stable} \\ |f'(x)| > 1 \Rightarrow \text{unstable} \end{cases} \]

\[ x = 0 \Rightarrow |f'(x)| = |r(1 - 2x)| = r, \begin{cases} r < 1 \Rightarrow \text{stable} \\ r > 1 \Rightarrow \text{unstable} \end{cases} \]
Class Book
  - Chapter 2, all sections
  - Chapter 7, sections 7.3 – Cellular Automata
  - Chapter 8, sections 8.1, 8.2, 8.3.10

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Papers and other materials
- Optional
    - Chapters 10, 11, 14 – Dynamics, Attractors and chaos