Exponential growth per Moore's Law
\[ N = 2300 \times 2^{0.5 \times (y-1975)} \]

Natural growth: Moore's Law soon will be seen to follow such a curve.
\[ N = \frac{M}{1 + e^{-\alpha (t - t_0)}} \]

EXPOSITIONAL GROWTH VERSUS NATURAL GROWTH
NO MORE LABS !!!
Exam Schedule

- 11595
  - Midterm
    - March 1st (Thursday)
      - Regular Class time
  - Final Exam
    - May 3rd (Thursday)
      - 7:15-9:15 p.m.
Readings until now

- **Lecture notes**
  - Posted online
    - [http://informatics.indiana.edu/rocha/i101](http://informatics.indiana.edu/rocha/i101)
      - *The Nature of Information*
      - *Technology*
      - *Modeling the World*
  - @ infoport
    - [http://infoport.blogspot.com](http://infoport.blogspot.com)

- **From course package**
    - Chapters 1, 4 (pages 1-12)
    - Chapter 10 (pages 13-17)
  - From Andy Clark’s book “*Natural-Born Cyborgs*”
    - Chapters 2 and 6 (pages 19 - 67)
  - From Irv Englander’s book “*The Architecture of Computer Hardware and Systems Software*”
    - Chapter 3: Data Formats (pp. 70-86)
    - Chapter 2: Classical Logic (pp. 87-97)
    - Chapter 3: Classical Set Theory (pp. 98-103)
    - Chapters 1-3 (pages 105-129)
    - OPTIONAL: Chapter 4 (pages 131-136)
    - Chapter 13 (pages 147-155)
    - Chapter 5 (pages 141-144)
  - Igor Aleksander, *Understanding Information Bit by Bit*
    - Pages 157-166
  - Ellen Ullman, *Dining with Robots*
    - Pages 167-172
Assignment Situation

Labs

Past

- Lab 1: Blogs
  - Closed (Friday, January 19): Grades Posted
- Lab 2: Basic HTML
  - Closed (Wednesday, January 31): Grades Posted
- Lab 3: Advanced HTML: Cascading Style Sheets
  - Closed (Friday, February 2): Grades Posted
- Lab 4: More HTML and CSS
  - Closed (Friday, February 9): Grades Posted
- Lab 5: Introduction to Operating Systems: Unix
  - Closed (Friday, February 16): Grades Posted
- Lab 6: More Unix and FTP
  - Closed (Friday, February 23): Grades Posted
- Lab 7: Logic Gates
  - Closed (Friday, March 9): Grades Posted
- Lab 8: Intro to Statistical Analysis using Excel
  - Closed (Friday, March 30): Grades Posted
- Lab 9: Data analysis with Excel (linear regression)
  - Closed (Friday, April 6): Grades Posted
- Lab 10: Simple programming in Excel and Measuring Uncertainty
  - April 12 and 13, Due April 20

Assignments

- Individual
  - First installment
    - Closed: February 9: Grades Posted
  - Second Installment
    - Past: March 2: Grades Posted
  - Third installment
    - Past: Grades Posted
  - Fourth Installment
    - Presented April 10th, Due April 20th

- Group
  - First Installment
    - Past: March 9th, graded
  - Second Installment
    - Past: April 6th Graded
  - Third Installment
    - Presented Thursday, April 12; Due Friday, April 27
Group Assignment

- Second Installment: Given the text of “Lottery of Babylon” by Jorge Luis Borges
  - Measures of central tendency and dispersion of letter frequency
  - Probability of a letter being a vowel
  - Probability of a letter being a consonant
  - Conditional probability of letters ‘e’ and ‘u’
    - \( P(e | \heartsuit) \) where \( \heartsuit \) is the letter occurring before ‘e’
    - \( P(u | \heartsuit) \) where \( \heartsuit \) is the letter occurring before ‘u’
    - Compute for all letters (not space)
    - Produce histogram of \( P(e | \heartsuit) \), for all \( \heartsuit \).
    - Produce histogram of \( P(u | \heartsuit) \), for all \( \heartsuit \).
    - Discuss the independence of ‘e’ and ‘u’ from other letters
  - Upload to Oncourse

\[
P(e | h) = \frac{\text{\( h \cap e \)}}{|h|} = \frac{\text{'he'}}{|h|}
\]

\[
P(e) = \frac{|e|}{N}
\]
Conditional Probability

- \[ P(B|A) = \frac{|A \land B|}{|A|} \]
- Multiplication Rule
  - \[ P(A \land B) = P(A) \cdot P(B|A) \]
- Two events A, B are \textit{independent} if the occurrence of one has \textit{no effect} on the probability of the occurrence of the other
  - \[ P(B|A) = P(B) \]
- Multiplication Rule
  - \[ P(A \land B) = P(A) \cdot P(B) \]
Lottery of Babylon (English)
La Lotería En Babilonia (Spanish)
Measures of central tendency and dispersion of letter frequency

Max Cutler and Marc Epstein

<table>
<thead>
<tr>
<th></th>
<th>English</th>
<th>Spanish</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Central Tendencies</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>431.61538</td>
<td>323.40625</td>
</tr>
<tr>
<td>Median</td>
<td>5611.5 (M)</td>
<td>5175 (L)</td>
</tr>
<tr>
<td>Mode</td>
<td>1436 (E)</td>
<td>1348 (E)</td>
</tr>
<tr>
<td><strong>Dispersion</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Range</td>
<td>1431</td>
<td>1348</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>377.1014799</td>
<td>380.1801661</td>
</tr>
<tr>
<td>Variance</td>
<td>142205.5262</td>
<td>144536.9587</td>
</tr>
</tbody>
</table>
Probability of letter being vowel or consonant

John Oglesby and Sarah Kepa

Probability of a Letter

- English: 37.81% (Probability of a letter being a vowel) + 62.19% (Probability of a letter being a consonant)

- Spanish: 45% (Probability of a letter being a consonant) + 55% (Probability of a letter being a vowel)
Lottery of Babylon (English)
| $P(e|a)$ | 0.002814819 | 0.986688889 |
| $P(e|b)$ | 0.923728479 | 0.085427493 |
| $P(e|c)$ | 0.212041988 | 0.099075799 |
| $P(e|d)$ | 0.192796717 | 0.460642912 |
| $P(e|e)$ | 0.987810412 | 0.022880868 |
| $P(e|f)$ | 0.058893968 | 0.149425287 |
| $P(e|g)$ | 0.104046248 | 0.079888889 |
| $P(e|h)$ | 0.440807089 | 0.290697874 |
| $P(e|i)$ | 0.484197279 | 0.098718351 |
| $P(e|j)$ | 0.4878 | 0.199398399 |
| $P(e|k)$ | 0.033080951 | 0.000000000 |
| $P(e|l)$ | 0.164559982 | 0.118356704 |
| $P(e|m)$ | 0.278646134 | 0.328972973 |
| $P(e|n)$ | 0.111056442 | 0.099087599 |
| $P(e|o)$ | 0.005912879 | 0.049792981 |
| $P(e|p)$ | 0.17820087 | 0.142877143 |
| $P(e|q)$ | 0.000000000 | 0.000000000 |
| $P(e|r)$ | 0.252288600 | 0.162016582 |
| $P(e|s)$ | 0.108524984 | 0.148994985 |
| $P(e|t)$ | 0.098748889 | 0.269148038 |
| $P(e|u)$ | 0.086920777 | 0.116299852 |
| $P(e|v)$ | 0.787604079 | 0.180202247 |
| $P(e|w)$ | 0.147819048 | 0.000000000 |
| $P(e|x)$ | 0.280792984 | 0.000000000 |
| $P(e|y)$ | 0.052179916 | 0.120689959 |
| $P(e|z)$ | 0.000000000 | 0.021789916 |
| \( P(u|a) \) | 0.0093444444 | 0.024602175 |
| \( P(u|b) \) | 0.0792079217 | 0.019079277 |
| \( P(u|c) \) | 0.0449026189 | 0.0 |
| \( P(u|d) \) | 0.0249042157 | 0.0 |
| \( P(u|e) \) | 0.0081934109 | 0.019180487 |
| \( P(u|f) \) | 0.0932858988 | 0.108478276 |
| \( P(u|g) \) | 0.0462427792 | 0.201858888 |
| \( P(u|h) \) | 0.0110758493 | 0.056189978 |
| \( P(u|i) \) | 0.0029434278 | 0.00192207 |
| \( P(u|j) \) | 0.0 | 0.889999999 |
| \( P(u|k) \) | 0.0 | 0.889999999 |
| \( P(u|l) \) | 0.0400848888 | 0.022886777 |
| \( P(u|m) \) | 0.0128979292 | 0.084745792 |
| \( P(u|n) \) | 0.0206560969 | 0.061390749 |
| \( P(u|o) \) | 0.1014829297 | 0.009240009 |
| \( P(u|p) \) | 0.0008699999 | 0.094980009 |
| \( P(u|q) \) | 0.5878889999 | 0.580952961 |
| \( P(u|r) \) | 0.0164289289 | 0.012911289 |
| \( P(u|s) \) | 0.0172898092 | 0.091189912 |
| \( P(u|t) \) | 0.0269235199 | 0.072892999 |
| \( P(u|u) \) | 0.0194747646 | 0.011290995 |
| \( P(u|v) \) | 0.0047619078 | 0.0 |
| \( P(u|w) \) | 0.0190494785 | 0.094462799 |
| \( P(u|x) \) | 0.0 | 0.0 |
| \( P(u|y) \) | 0.0190494785 | 0.094462799 |
| \( P(u|z) \) | 0.0 | 0.0 |

\( P_E('u') = 0.03 \)

\( P_S('u') = 0.04 \)
Luis M. Rocha and Santiago Schnell

Marcus Bigbee & Brandon Smith (‘e’ Spanish)

\[ P(\text{‘e’} | \text{‘?’}) \]

\[ P(\text{‘e’}) = 0.13 \]
Marcus Bigbee & Brandon Smith (‘u’ Spanish)

\[ P(‘u’) = 0.04 \]
P('e') = 0.13
Craig Bauer & Chris Kremser ('u' English)

P('u') = 0.03
Group Assignment

Third Installment

Given any text such as the library of babylon or Funes, the memorious

Create a database model and a relational database instance using Microsoft Access to store the data and conclusions from previous installments

- Use the entity-relationship model
- Examples of items that should appear
  - Title, author, language, publication date
  - Frequency/probability of each letter
  - Conditional probabilities for letters ‘e’ and ‘u’ (as produced in installment 2)
  - Positively and negatively dependent letters
- Use at least 4 texts

Due on April 27th, 2005
Upload to Oncourse
Individual Assignment – Part IV

- Step by step analysis of “dying” squares
  - 4th Installment
    - Presented: April 10th
    - Due: April 20th

- Use inductive and deductive reasoning
  - To uncover the algorithm in each quadrant
    - Build from inductive knowledge accumulated so far
Summary of Black Box

- **Quadrant 1**
  - At the random initial state
    - All numbers have equal probability of being initially present
    - But the probability of changes are different
  - In Any State
    - Any number changes depending on its neighbors
    - It ‘gravitates’ towards the smallest number that it ‘sees’ most often.
    - Odd and Even numbers do not show different behavior
- **What is the Algorithm?**
Summary of Black Box

- **Quadrant 3**
  - At the random initial state
    - All numbers have equal probability of being initially present
    - But the probability of changes are different
  - **In Any State**
    - 0 can only change to 0
    - 5 can only change to 5 or 0
    - Even digits always change to even digits
    - Odd digits could change to any other digit

- **What is the Algorithm?**

<table>
<thead>
<tr>
<th>n(i)</th>
<th>p(i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.27</td>
</tr>
<tr>
<td>1</td>
<td>0.04</td>
</tr>
<tr>
<td>2</td>
<td>0.12</td>
</tr>
<tr>
<td>3</td>
<td>0.04</td>
</tr>
<tr>
<td>4</td>
<td>0.12</td>
</tr>
<tr>
<td>5</td>
<td>0.09</td>
</tr>
<tr>
<td>6</td>
<td>0.12</td>
</tr>
<tr>
<td>7</td>
<td>0.04</td>
</tr>
<tr>
<td>8</td>
<td>0.12</td>
</tr>
<tr>
<td>9</td>
<td>0.04</td>
</tr>
</tbody>
</table>

1. $0 \rightarrow 0$
2. $\{5\} \rightarrow \{0, 5\}$
3. $\{2, 4, 6, 8\} \rightarrow \{0, 2, 4, 6, 8\}$
4. $\{1, 3, 7, 9\} \rightarrow \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
Summary of Black Box

- **Quadrant 2**
  - **At the random initial state**
    - All numbers have equal probability of being initially present
    - But the probability of changes are different
  - **In Any State**
    - 0 can only change to 0
    - 5 can only change to 5 or 0
    - Even digits always change to even digits
    - Odd digits could change to any other digit

- **What is the Algorithm?**

  1. 0 → 0
  2. \{ 5\} → \{0, 5\}
  3. \{2, 4, 6, 8\} → \{0, 2, 4, 6, 8\}
  4. \{1, 3, 7, 9\} → \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}
# Possible Operations Q2 and Q3

<table>
<thead>
<tr>
<th>Operator</th>
<th>Meaning</th>
<th>Excel</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>()</td>
<td>Brackets, grouping</td>
<td>()</td>
<td>( y = (a + b) \times (c + d) )</td>
</tr>
<tr>
<td>*</td>
<td>Multiplication</td>
<td>*</td>
<td>( i = j \times k )</td>
</tr>
<tr>
<td>+</td>
<td>Add</td>
<td>+</td>
<td>( i = i + 1 )</td>
</tr>
<tr>
<td>-</td>
<td>Subtract</td>
<td>-</td>
<td>( i = j - 3.2 )</td>
</tr>
<tr>
<td>/</td>
<td>Real division</td>
<td>/</td>
<td>( i = 8 / 5 = 1.6 )</td>
</tr>
<tr>
<td>div</td>
<td>Integer division</td>
<td>Quotient (a, b)</td>
<td>( i = 8 / 5 = 1 )</td>
</tr>
<tr>
<td>Mod, %</td>
<td>remainder</td>
<td>Mod (a, b)</td>
<td>( i = 8 \mod 5 = 3 )</td>
</tr>
<tr>
<td>ROUND</td>
<td>Rounds</td>
<td>ROUND (a, d)</td>
<td>( i = \text{ROUND}(3.67, 0) = 4 )</td>
</tr>
<tr>
<td>INT</td>
<td>Integer Part</td>
<td>INT</td>
<td>( i = \text{INT}(3.67) = 3 )</td>
</tr>
<tr>
<td>rand</td>
<td>Random number</td>
<td>Rand()</td>
<td>( i = \text{rand}(n) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RandBetween(a, b)</td>
<td></td>
</tr>
</tbody>
</table>
Tip for Individual Assignment

- Quadrant Q
  - There are 100 cells in each 10x10 quadrant
    - \( C = 1 \ldots 100 \)
  - Each cell can take one of 10 colors
    - \( V(C) = 0 \ldots 9 \)
      - is the value of the cell
      - This is the state cell \( C \) is in
  - Random initialization of quadrant Q at cycle 1
    - For \( c = 1 \) to 100 do
      - \( V(C) \leftarrow \text{randbetween}(0,9) \) \{random number 0 to 9\}
    - EndFor
    - Cycle \( \leftarrow 1 \)
  - Run for Number of cycles
    - \( n \leftarrow \) Input dialog
    - For \( k = 1 \) to \( n \) do
      - Cycle \( \leftarrow \) cycle+1
        - \{Pick random cell\}
      - \( C \leftarrow \text{randbetween}(1,100) \)
        - \{Update the value of the cell \( NOT \) THE REAL THING\}
      - \( V(C) \leftarrow ((V(C) \times \text{randbetween}(0,9)) \div 2) - 5 \times X \)
    - EndFor
  - \( X \) may be a hidden variable
    - \( X \leftarrow ??? \)
The Modeling Relation

Hertz' Modeling Paradigm

- **Modeling**
  - Compute hypothesis
- **Rules from Inductive and Deductive Analysis**
  - From Data analysis
  - Produce Conclusions
Types of Problems

- Algorithms are for Solving Problems
- Types of Problems
  - Search
    - Find an X in input satisfying property Y
    - Find a prime number in a random sequence of numbers
  - Structuring Problems
    - Transform the input to satisfy property Y
      - Sort a random sequence of numbers
  - Construction Problems
    - Build an X satisfying property Y
      - Generate a random sequence of numbers with a given mean and standard deviation
  - Optimization Problems
    - Find the best X satisfying property Y
      - Find the largest prime number in a given sequence
  - Decision Problems
    - Decide whether the input satisfies property Y
      - Is the input number a prime number
  - Adaptive Problems
    - Maintain property Y over time
      - Grow a sequence of numbers such that there are always m prime numbers with a given mean and standard deviation
Problem Difficulty

- **Conceptually Hard Problem**
  - No algorithm exists to solve the problem

- **Analytically Hard Problem**
  - An algorithm exists to solve the problem, but we don’t know how long it will take to solve every instance of the problem

- **Computationally Hard Problem**
  - An algorithm exists to solve the problem, but relatively few instances take millions of years to solve
    - Problems we know to be
    - problems we suspect to be

- **Computationally unsolvable Problem**
  - No algorithm can exist to solve the problem
Hanoi Problem

- Invented by French Mathematician Édouard Lucas in 1883

- At the Tower of Brahma in India, there are three diamond pegs and sixty-four gold disks. When the temple priests have moved all the disks, one at a time preserving size order, to another peg the world will come to an end.

- If the priests can move a disk from one peg to another in one second, how long does the World have to exist?
Solving the Hanoi Problem

- Solve for the smallest instances and then try to generalize

- \( N=2 \)

- \( N=3 \)

Use Hanoi_2 (H2) as building block (of 3 moves)
H3 uses H2 twice, plus 1 move of the largest disk
Hanoi Problem for $n$ disks

- Algorithm to move $n$ disks from $A$ to $C$
  - Move top $n-1$ disks from $A$ to $B$
  - Move biggest disk to $C$
  - Move $n-1$ disks on $B$ to $C$
- Recursion
  - Until $H2$

Use Hanoi_2 ($H2$) as building block (of 3 moves)
$H3$ uses $H2$ twice, plus 1 move of the largest disk

An Algorithm that uses itself to solve a problem
Pseudocode for Hanoi Problem

- **Hanoi** *(Start, Temp, End, n)*
  - If \( n = 1 \) then
    - Move *Start’s* top disk to *End*
  - Else
    - **Hanoi** *(Start, End, Temp, n-1)*
    - Move *Start’s* top disk to *End*
    - **Hanoi** *(Temp, Start, End, n-1)*
Computational Complexity

- Resources required during computation of an algorithm to solve a given problem
  - Time
    - how many steps does it take to solve a problem?
  - Space
    - how much memory does it take to solve a problem?

- The Hanoi Towers Problem
  - $f(n)$ is the number of times the HANOI algorithm moves a disk for a problem of $n$ disks
    - $f(1)=1, f(2)=3, f(3)=7$
    - $f(n)=f(n-1) + 1 + f(n-1) = 2 \times f(n-1) + 1$
  - Every time we add a disk, the time to compute is at least double
    - $f(n) = 2^n - 1$

$2^{10} (KILO) = 1,024$
$2^{20} (MEGA) = 1,048,576$
$2^{30} (GIGA) = 1,073,741,824$
$2^{40} (TERA) = 1,099,511,627,776$
$2^{64} = 18,446,744,073,709,551,616$

585 billion years in seconds!!!!!!!!

Earth: 5 billion years
Universe: 15 billion years
Fastest Computer: 135.5 teraflops - 135.5 trillion calculations a second (aprox $2^{47}$ moves a second)
$2^{17}$ s needed = 36 hours

217 s needed = 36 hours

Luis M. Rocha and Santiago Schnell
IBM Blue Gene/L

"FLOPS"
(FLoating Point Operations Per Second)

MDGRAPE-3

Fastest Computer (June 2006): 1 petaflop !!! – 1 quadrillion calculations per second --- MDGRAPE-3 @ Riken, Japan --- approx $2^{14}$ s needed = 4.6 hours for Hanoi problem (assuming one disk change per operation)

Fastest Computer (Late 2005): 280.6 teraflops - 280.6 trillion calculations a second --- Approaching petaflops: 3 petaflops in late 2006????

Fastest Computer (Early 2005): 135.5 teraflops - 135.5 trillion calculations a second --- Approaching petaflops: $2^{50}$
Bremermann's Limit

Physical Limit of Computation

- Hans Bremermann in 1962
- “no data processing system, whether artificial or living, can process more than $2 \times 10^{47}$ bits per second per gram of its mass.”

- Based on the idea that information could be stored in the energy levels of matter
- Calculated using Heisenberg's uncertainty principle, the Hartley measure, Planck's constant, and Einstein's famous $E = mc^2$ formula

- A computer with the mass of the entire Earth and a time period equal to the estimated age of the Earth
  - would not be able to process more than about $10^{93}$ bits
- *transcomputational problems*
Transcomputational Problems

- A system of $n$ variables, each of which can take $k$ different states
  - $k^n$ possible system states
  - When is it larger than $10^{93}$?

<table>
<thead>
<tr>
<th>$k$</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>308</td>
<td>194</td>
<td>154</td>
<td>133</td>
<td>119</td>
<td>110</td>
<td>102</td>
<td>97</td>
<td>93</td>
</tr>
</tbody>
</table>

- Pattern Recognition
  - Grid of $n = q^2$ squares of $k$ colors
  - Blackbox: $10^{100}$ possible states!
  - The human retina contains a million light-sensitive cells

- Large scale integrated digital circuits
  - $K=2$ (bits): a circuit with 308 inputs and one output!

- Complex Problems need simplification!
What happens to Moore’s law?

Exponential growth per Moore’s Law

\[ N = 2300 \times 2^{0.5(y-1971)} \]

Natural growth: Moore’s Law soon will be seen to follow such a curve.

\[ N = \frac{M}{1 + e^{-(at+b)}} \]

EXPONENTIAL GROWTH VERSUS NATURAL GROWTH
Next Class!

- Topics
  - Databases and SQL

- Readings for Next week
  - @ infoport
  - From course package
    - Igor Aleksander, "Understanding Information Bit by Bit"
      - Resources tab in onCourse.
    - Ellen Ullman, "Dining with Robots"
      - Resources tab in onCourse.

- No More Labs!!!!!!